The NKM with a supply shock: Matrix Form

Macroeconomics (M8674), May 2025

Vivaldo Mendes, ISCTE

vivaldo.mendes@iscte-iul.pt

The New Keynesian Model with a supply shock

- There is also a shock to the r_t^n that vanishes from the reduced form of the model

$$
\begin{aligned}
IS: &\quad \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \\
AS: &\quad \pi_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} + s_t \\
MP: &\quad i_t = \pi_t + r_t^n + \phi_\pi \pi_t + \phi_y \hat{y}_t \\
\text{Shocks}: &\quad r_t^n = \rho_r \cdot r_{t-1}^n + \varepsilon_t^r \, , \qquad s_t = \rho_s \cdot s_{t-1} + \varepsilon_t^s
\end{aligned}
$$

- $\{i, r_t^n, \hat{y}, \pi, s_t, \varepsilon_t\}$: nominal interest rate, natural real interest rate, output-gap, inflation rate, supply shock, and a random disturbance.
- $\{\sigma, \kappa, \beta, \phi_\pi, \phi_y, \pi_t^*, \rho\}$ are parameters
- Forward-looking variables: $\hat{y}_t, \pi_t$ // Backward-looking variables: r_t^n, s_t
- Static variables: i_t

Simplifying the IS curve

- Notice that the model can be reduced to three equations by inserting the MP curve into the IS curve.

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}[i_t - \mathbb{E}_t \pi_{t+1} - r_t^n] \qquad (IS)$$

$$\downarrow \qquad \nwarrow \; i_t = \pi_t + r_t^n + \phi_\pi \pi_t + \phi_y \hat{y}_t \qquad (MP)$$

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left[\pi_t + r_t^n + \phi_\pi \pi_t + \phi_y \hat{y}_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right]$$

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left[(1 + \phi_\pi)\pi_t + \phi_y \hat{y}_t - \mathbb{E}_t \pi_{t+1}\right]$$

$$\frac{1}{\sigma}\mathbb{E}_t \pi_{t+1} + 1\mathbb{E}_t \hat{y}_{t+1} = \left(\frac{1 + \phi_\pi}{\sigma}\right)\pi_t + \left(\frac{\phi_y}{\sigma} + 1\right)\hat{y}_t$$

3 equations vs 3 unknowns

- The three equations

$$\frac{1}{\sigma}\mathbb{E}_t\pi_{t+1} + 1\mathbb{E}_t\hat{y}_{t+1} = \left(\frac{1+\phi_\pi}{\sigma}\right)\pi_t + \left(\frac{\phi_y}{\sigma} + 1\right)\hat{y}_t \tag{IS}$$

$$\beta\mathbb{E}_t\pi_{t+1} = \pi_t - \kappa\hat{y}_t - s_t \tag{AS}$$

$$s_{t+1} = \rho_s s_t + \varepsilon^s_{t+1} \tag{Supply shock}$$

- The three unknowns
  - π_t , $\hat{y}_t$, s_t, for $t = 1, \ldots, n$

Matrix representation

$$1s_{t+1} + 0\mathbb{E}_t\pi_{t+1} + 0\mathbb{E}_t\hat{y}_{t+1} = \rho_s s_t + 0\pi_t + 0\hat{y}_t + 1\varepsilon^s_{t+1}$$

$$0s_{t+1} + \beta\mathbb{E}_t\pi_{t+1} + 0\mathbb{E}_t\hat{y}_{t+1} = -1s_t + 1\pi_t - \kappa\hat{y}_t + 0\varepsilon^\pi_{t+1}$$

$$0s_{t+1} + \frac{1}{\sigma}\mathbb{E}_t\pi_{t+1} + 1\mathbb{E}_t\hat{y}_{t+1} = 0s_t + \left(\frac{1+\phi_\pi}{\sigma}\right)\pi_t + \left(\frac{\phi_y}{\sigma} + 1\right)\hat{y}_t + 0\varepsilon^y_{t+1}$$

..

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 1/\sigma & 1 \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} s_{t+1} \\ \mathbb{E}_t\pi_{t+1} \\ \mathbb{E}_t\hat{y}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_s & 0 & 0 \\ -1 & 1 & -\kappa \\ 0 & \left(\frac{1+\phi_\pi}{\sigma}\right) & \left(\frac{\phi_y}{\sigma} + 1\right) \end{bmatrix}}_{\mathcal{B}} \begin{bmatrix} s_t \\ \pi_t \\ \hat{y}_t \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} \varepsilon^s_{t+1} \\ \varepsilon^\pi_{t+1} \\ \varepsilon^y_{t+1} \end{bmatrix}$$

The model is ready for the computer

- Use the notebook provided in item 7 in the course's website.